

Math 20550 - Summer 2016  
Cross Product and Lines Worksheet  
June 15, 2016

**Problem 1.** Compute the cross product

$$\langle 1, 3, -2 \rangle \times \langle -1, 5, 7 \rangle$$

**Problem 2.** Compute the cross product

$$(3\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})$$

**Problem 3.** Compute  $(\hat{i} \times \hat{j}) \times \hat{j}$ .

**Problem 4.** Compute  $\hat{i} \times (\hat{j} \times \hat{j})$ .

**Problem 5.** If  $\vec{a} \times \vec{b} = \langle 3, -7, -2 \rangle$ , then what is  $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ ?

**Problem 6.** Identify each of the following expressions as a vector, scalar, or nonsense:

(1)  $(\vec{a} \times \vec{b}) \times \vec{c}$

(2)  $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$

(3)  $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

**Problem 7.** Find a unit vector perpendicular to  $\langle 2, 1, -3 \rangle$  and  $\langle 1, 0, 1 \rangle$ .

**Problem 8.** Give an example of vectors  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  which shows that

$$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c}).$$

**Problem 9.** Find the area of the parallelogram with vertices  $(1, 1)$ ,  $(3, 2)$ ,  $(1, 3)$ , and  $(-1, 2)$ .

**Problem 10.** Find the area of the triangle with vertices  $(1, 0, 1)$ ,  $(0, 2, 3)$ , and  $(-1, 5, -2)$ .

**Problem 11.** Find the volume of the parallelepiped spanned by the vectors  $\vec{a} = \langle 3, -1, 0 \rangle$ ,  $\vec{b} = \langle -2, 0, 1 \rangle$ , and  $\vec{c} = \langle 1, -2, 4 \rangle$ .

**Problem 12.** Brian is trying to open a jar of grape jelly. The radius of the lid of the jar is 5cm. If Brian imparts 15N of force tangent to the edge of the lid to open the jar, how many N · m, and in what direction, is the resulting torque?

**Problem 13.** Rewrite the following line in vector form

$$\frac{x-1}{2} = \frac{y+2}{4} = \frac{z-4}{-4}.$$

**Problem 14.** Rewrite the following line as symmetric equations

$$\vec{\ell}(t) = \langle 2-t, 3t, 9 \rangle.$$

**Problem 15.** Find parametric equations for the line passing through the point  $(-2, 0, 3)$  parallel to the vector  $\langle 3, 1, 5 \rangle$ .

**Problem 16.** Find symmetric equations for the line passing through the point  $(0, 0, 0)$  which is parallel to the line  $\vec{\ell}(t) = \langle 1-3t, -1-2t, 3+t \rangle$ .

**Problem 17.** Find the point of intersection between the lines

$$\begin{aligned} L_1 : x &= 4t + 2, & y &= 3, & z &= -t + 1 \\ L_2 : x &= 2s + 2, & y &= 2s + 3, & z &= s + 1 \end{aligned}$$

**Problem 18.** Find the angle of intersection of the lines  $L_1$  and  $L_2$  in Problem 17.

**Problem 19.** Find a line which passes through the point of intersection of the lines in Problem 17 which is perpendicular to both lines  $L_1$  and  $L_2$ .

**Problem 20.** Determine whether the lines

$$\begin{aligned} L_1 : x &= 6 - 3t, & y &= -2 + 2t, & z &= 5 + 4t \\ L_2 : x &= 10 - 6t, & y &= 3 + 4t, & z &= 7 + 8t \end{aligned}$$

are identical, parallel, skew, or intersecting.

**Problem 21.** Determine whether the lines

$$\frac{x}{3} = \frac{y - 2}{-1} = z + 1$$

and

$$\frac{x - 1}{4} = y + 2 = \frac{z + 3}{-3}$$

are identical, parallel, skew, or intersecting.

**Problem 22.** Give the equation of a line perpendicular to

$$\vec{\ell}(t) = \langle 1 + t, 2 + 2t, 3 + 3t \rangle.$$

**Problem 23.** Find a line perpendicular to the line in Problem 22.

**Problem 24.** Find a line perpendicular to the lines in Problems 22 and 23.

**Problem 25.** Among the lines in Problems 22, 23, and 24, do any of them intersect?

**Problem 26.** Show that

$$\|\vec{a} \times \vec{b}\|^2 = \|\vec{a}\|^2 \|\vec{b}\|^2 - (\vec{a} \cdot \vec{b})^2$$